The Number Line Connection to the Common Core

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Common Core Means Change

Today our very survival depends on our ability to stay awake, to adjust to new ideas, to remain vigilant and to face the challenge of change.

Martin Luther King, Jr.

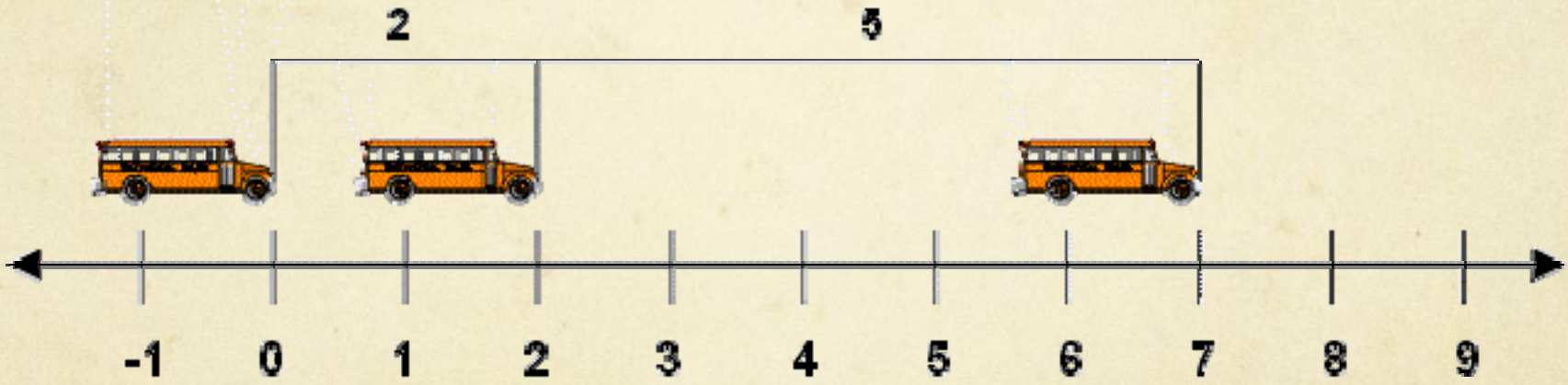
Number Lines and the CCSS

The number line serves as a visual /physical model to represent the counting numbers and constitutes an effective tool to develop estimation techniques, as well as a helping instrument when solving word problems.

CA Math Project

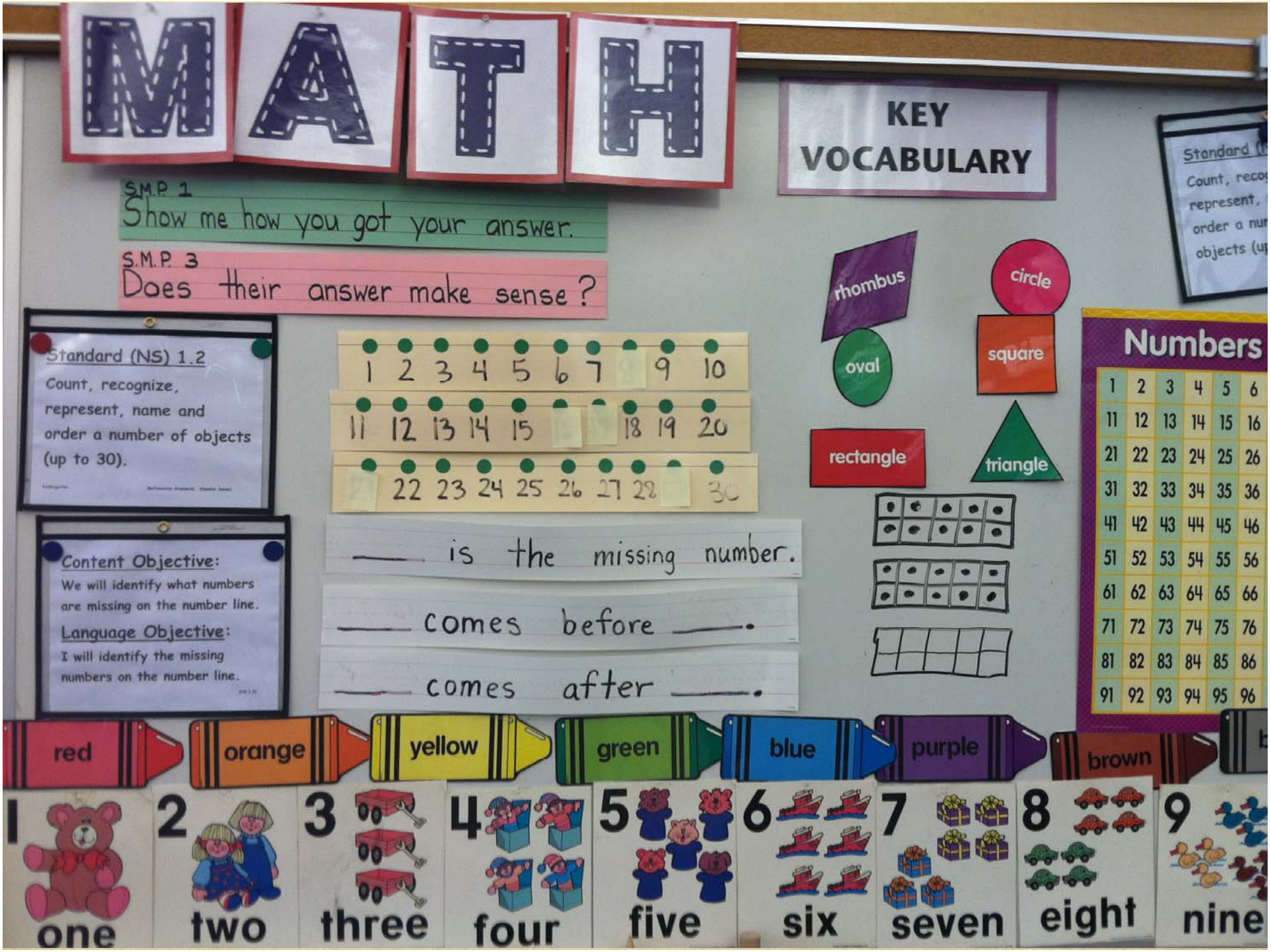
Essential Question

How can the number line be used to help students make sense of numbers?

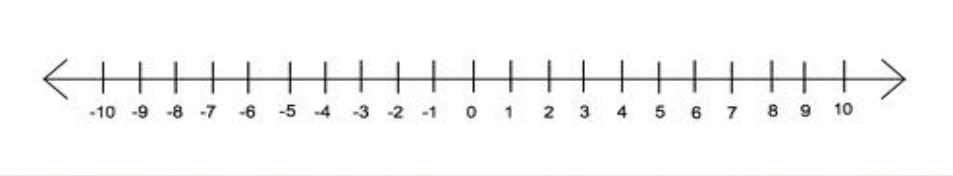


Standards for Mathematical Practice

1. **Make sense of problems** and persevere in solving them.
2. Reason abstractly and quantitatively.
3. **Construct viable arguments** and critique the reasoning of others.
4. Model with mathematics
5. **Use appropriate tools strategically.**
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.


MATH
KEY VOCABULARY
S.M.P. 1
Show me how you got your answer.
S.M.P. 3
Does their answer make sense?
Standard (NS) 1.2
Count, recognize, represent, name and order a number of objects (up to 30).
Content Objective:
We will identify what numbers are missing on the number line.
Language Objective:
I will identify the missing numbers on the number line.
1 2 3 4 5 6 7 9 10
11 12 13 14 15 18 19 20
22 23 24 25 26 27 28 30
___ is the missing number.
___ comes before ___.
___ comes after ___.
rhombus
circle
oval
square
rectangle
triangle
Numbers
1 2 3 4 5 6
11 12 13 14 15 16
21 22 23 24 25 26
31 32 33 34 35 36
41 42 43 44 45 46
51 52 53 54 55 56
61 62 63 64 65 66
71 72 73 74 75 76
81 82 83 84 85 86
91 92 93 94 95 96
red
orange
yellow
green
blue
purple
brown
1 one
2 two
3 three
4 four
5 five
6 six
7 seven
8 eight
9 nine

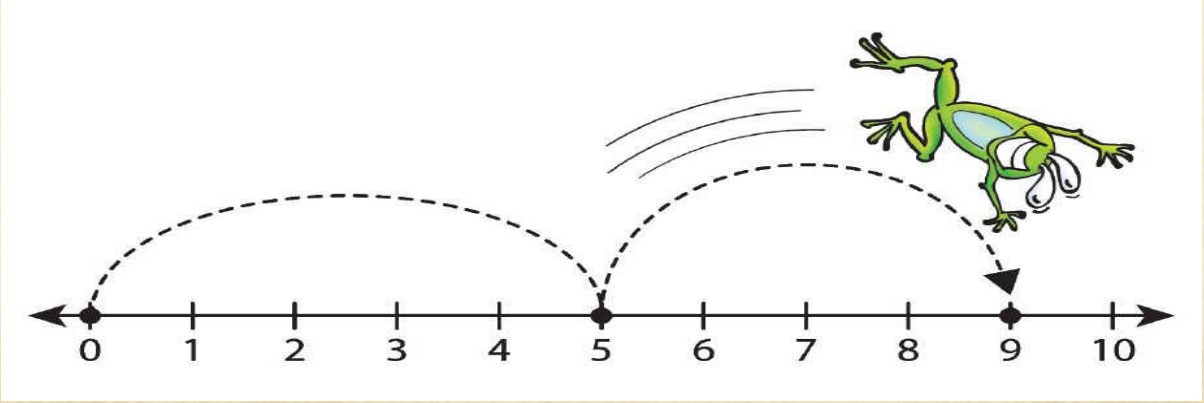
What is a Number Line?



- A line with points labeled by numbers
- A simple way to interpret whole numbers, negative numbers, decimals and fractions as part of one overall system
- A tool to organize thinking about number and make connections between arithmetic and geometry
- A visual model to represent the **distance** from zero to a labeled point**, not the point** itself

Correct Use of Number Lines

- Use with jumps to show that the jump is linked to the distance and not the points on the line.
- The jumps must be explicitly marked to make a connection to the distance moved rather than just pointing to the numbers.

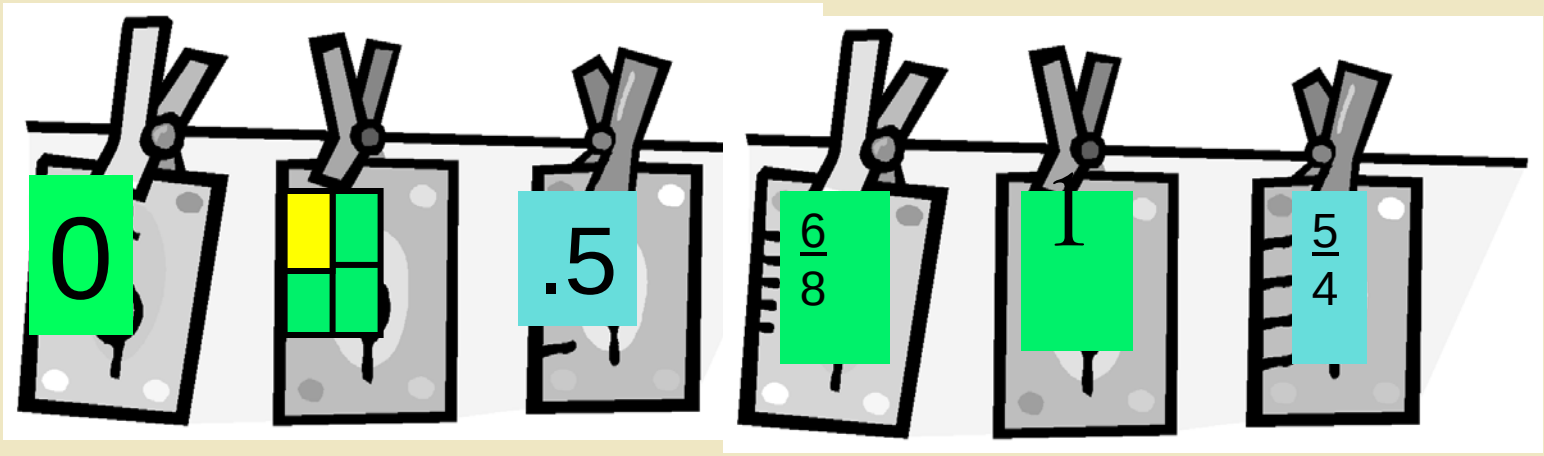


Brain Research

- Humans possess an innate number line and see numbers as points on a line.
- Mental number lines don't place numbers evenly.
  - The farther we go on our mental number line, the closer together the numbers appear.
- We have good innate number sense of small positive integers.
  - For other numbers (fractions, irrational numbers, negative numbers) we construct number line models to provide understanding.

Big Ideas

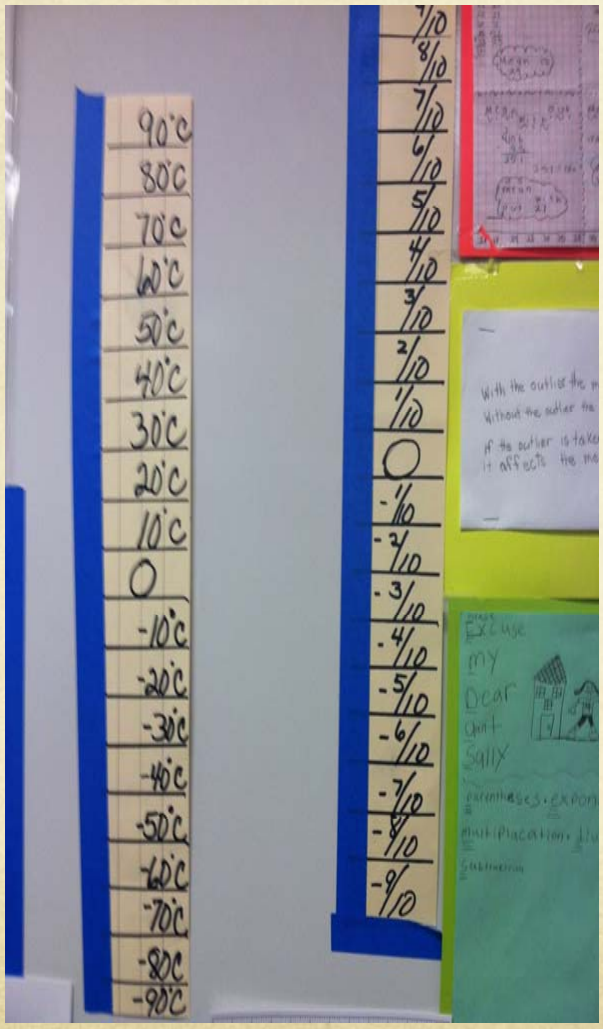
- Visual Representation – model of distance and/or direction
  - Both vertical and horizontal
- Goes on in both directions forever
- Infinite number of points between any two points
- Has symmetry, models ordering and magnitude of numbers
- Great starting point for showing multiple representations
- Emphasized in the Common Core Standards
  - Builds from TK to Calculus



Shifts in Pedagogy

- Make the shift toward teaching for understanding rather than telling students how to solve a problem.
- Practice asking open ended questions rather than telling students how to do the math.
- Give yourself the time and the freedom to be a learner along with your students.
- Be prepared to grow in your own understanding of mathematics.

Begin with Positives and Negatives



Watch the tutorial, *Curly and Bruce Play Tug of War.*

When do you think we should teach negative integers?

http://www.sophia.org/integers-from-10-to-10/integers-from-10-to-10--9-tutorial

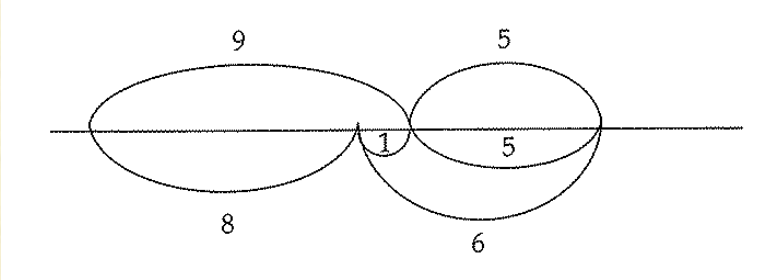
Number Line Riddles

1. I am two more than five and three less than ten.
2. Double 3 and add 1 more.
3. I'm halfway between -6 and 8.
4. If you add 4 to me, you would be at 0.
5. Start at -3 and count up 9.
6. Count down 5 spaces from 3.

Addition on the Number Line

- The number line is an important tool for modeling strategies for addition and subtraction.

9+5 = 8 + 6



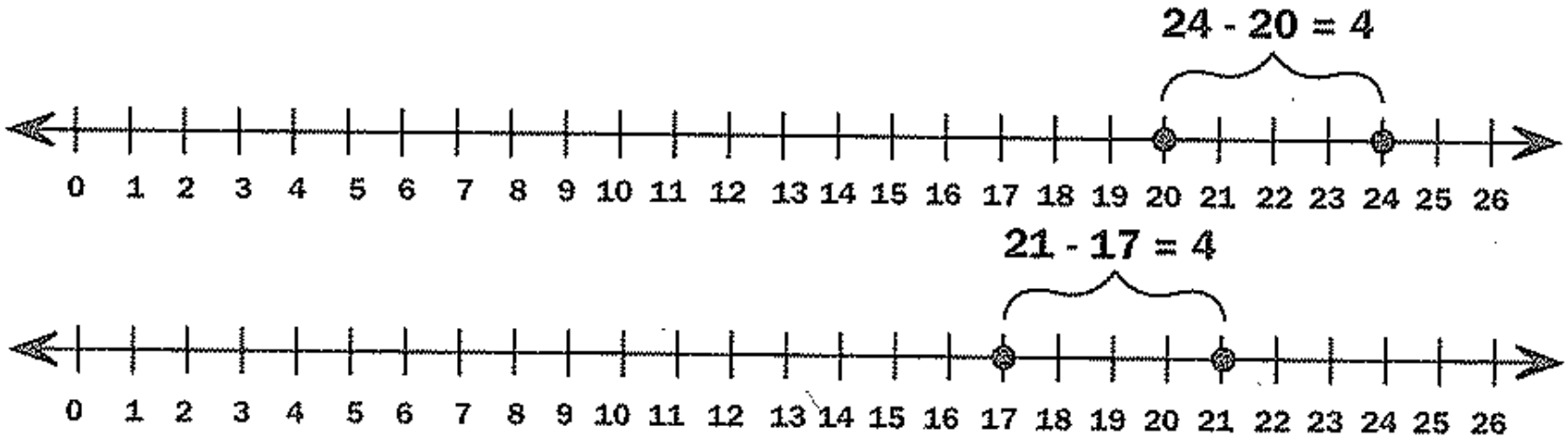
- Develops conceptual understanding and computational strategies.
- Open number lines allow students to start and end at any number.

Addition-Subtraction Connection

42-17 is the same as thinking about how much to add to 17 to make 42.

- Strategies include:
  - **Counting up by tens**
    - 17 + 10 = 27, 27 + 10 = 37, 37 + 3 = 40, 40+ 2 = 42
    - 10 + 10 + 3 + 2= 25
  - **Jumping to the next friendly number**
    - 17 to 20 and 20 to 40 and 40 to 42

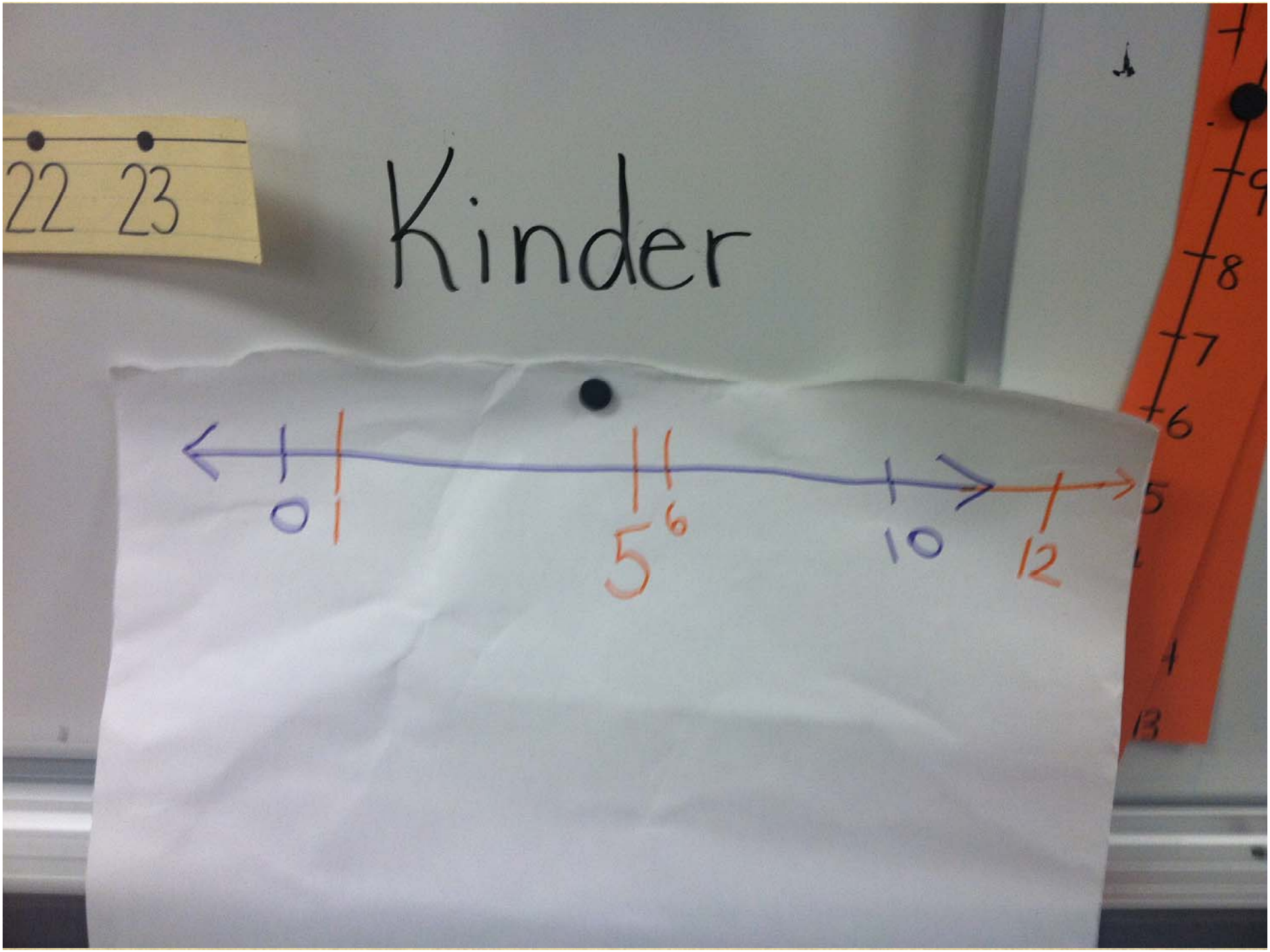
Find the Difference



Where Do I Go?

1. Ask students where to place the zero? Where should I place the ten? Can you show me where the 5 belongs? Where should I place 8?

2. Place the zero and ten on the number line. Place two empty cards on the line, one midpoint and one where you might see the two. What numbers go in each box?


22 23
Kinder
0 1
5 6
10
12

How Far Away?

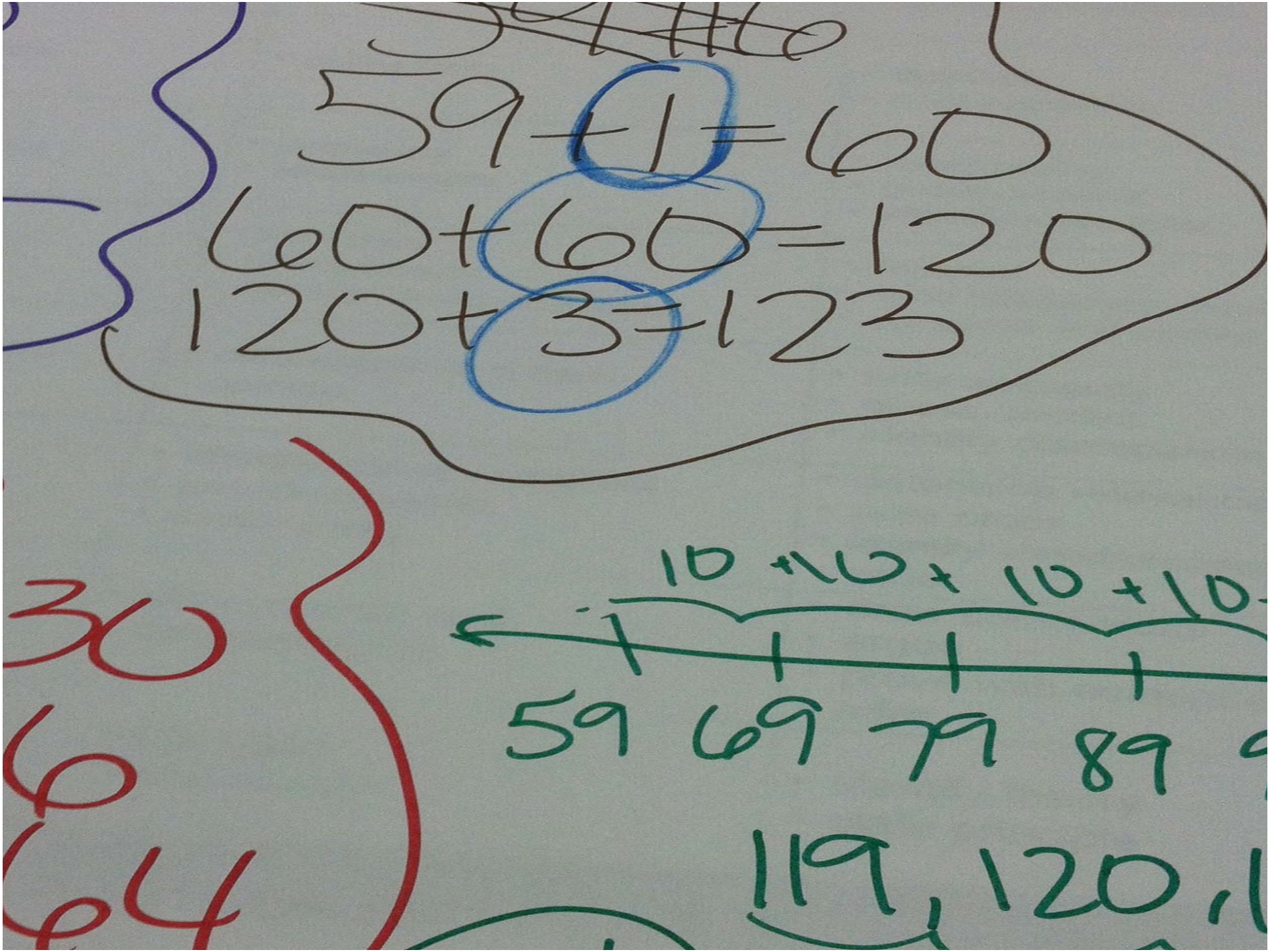
1. Place five numbers on the number line
2. Ask the following questions:
   a. Which two numbers are closest together? How close?
   b. How far is it from 16 to 20?
   c. What number card would be halfway between 4 and 16?
   d. Is it further to go from 4 to 7 or from 16 to 20?
   e. What number is less than 16 but greater than 4?

Doubles

1. There are four people at your table. Without looking, think about how many shoes are under the table?

2. Fill in a chart to help determine how many shoes if there are 5,6, 12 or *n* people at the table.

3. Use the number line to help. Place a string between the numbers 0 – 5. Use that same string to represent the distance between 5 and 10.

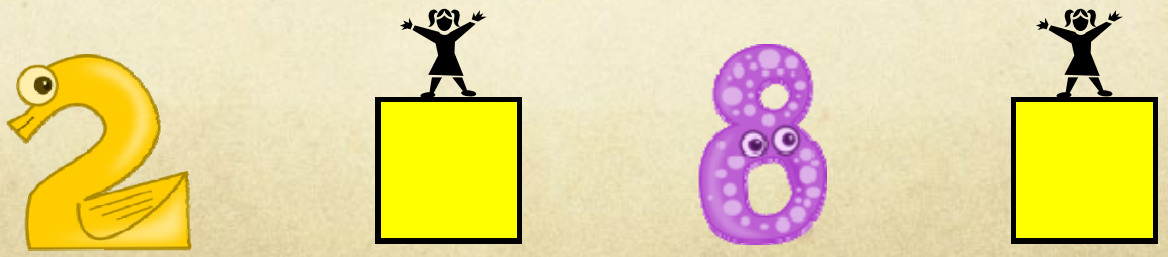

59+1=60
60+60=120
120+3=123
10 +10 + 10 +10
59 69 79 89
119, 120,
30
6
64

What's My Number?

1. Put a 2 on the left side of the number line. Place three empty cards to the right of the 2. Place an 8 in the middle box and ask what the other two cards will be.

2. Place a 4 to the left and a 16 to the right. What numbers go on the two cards in between?

3. Place a 0 in the middle and a 2 to the right. What number goes on the empty card to the left?



One Minute Challenge

1. Student A tosses the number cube and calls out the number. Student B counts that number of beans and puts them to the side.

2. At the end of one minute, each student guesses how many beans have been accumulated. Student A counts the beans and writes that number on a card.

3. Reverse roles and repeat steps 1-3.

4. Place numbers at the appropriate spot on the floor number line .

5. For older students, ask them to find the difference between the two numbers using various number line strategies.

* Challenge students to count the most beans in a minute

Greater Than, Less Than, Equal To Game

1. Team A spins the number spinner and places a foot paper clip on the number line to mark that number.

2. Another member of Team A spins the greater than, less than, equal to spinner.

3. Team A looks at their cards to find a number that fits. That card is placed on the table in the correct proximity to the first number.

4. If Team A doesn't have a number, play passes to Team B who spins for a new number and moves the footed paper clip to that number.

5. The first team to use up their cards is the winner.

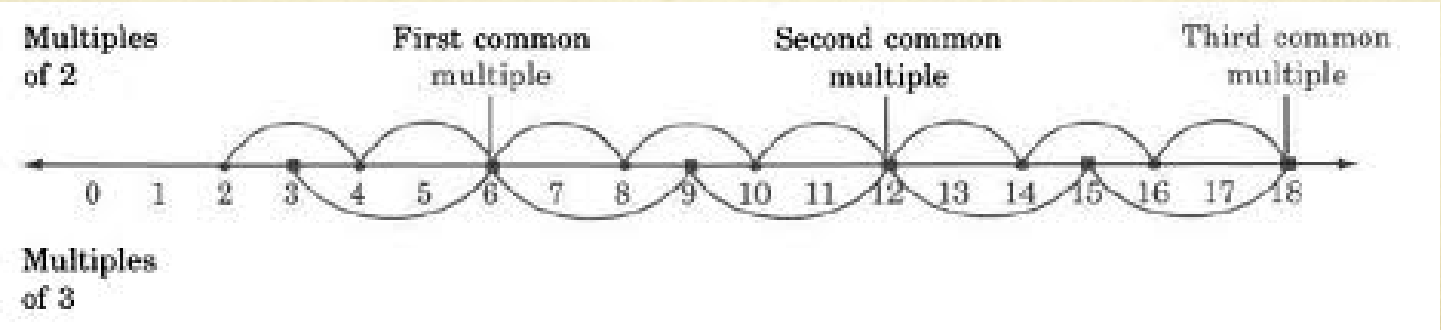
Skipping on a Number Line

Use frogs to "skip jump" on the number line

1. Skip jump by 2's and place a red dot above those numbers

1. Skip jump by 5's and place a green dot above the multiples

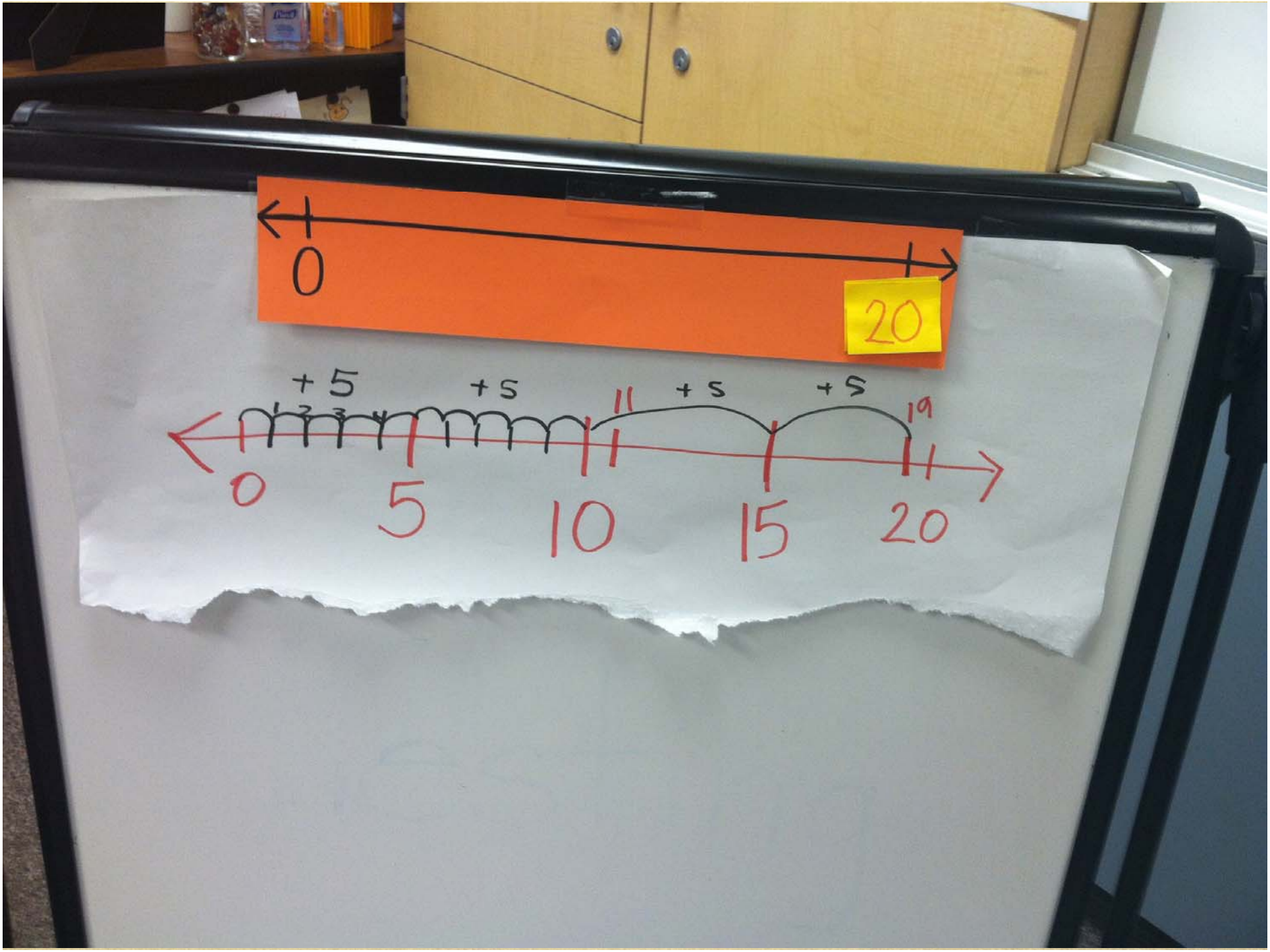
1. Skip jump by 10's and place a blue dot above the multiples




Ii
Jj
Kk
Ll
Mm
Nn
f
ph
fire
g
guitar
h
hippo
i
insect
jump
kite
lemon
map
net
5
10
15
20
25
30
Today is Wednesday, March 21, 2012
Cause → Effect
Different
like
assion

Target Practice

1. Start at zero and take two jumps to make it to 15.
   a. Show your jumps by marking them on the number line.
2. Go from 0 to 6 in 2 jumps, in 3 jumps
3. Go from 0 to 8 in two jumps, in 2 jumps
4. Go from 0 to 12 in ? jumps


0
20
+5
+5
+5
+5
11
19
0
5
10
15
20

Sharing Licorice

1. Place a long rope of licorice on freezer paper.
2. Label the left side of the freezer paper under the licorice rope 0 and the right side 1.
3. Show how you would share the licorice with one friend.
4. Write that number on your freezer paper.
5. Show how to share with 3 friends, 4 friends.

Kindergarten CC Standards

Counting and Cardinality (CC)

- **K.CC.1** - Count to 100 by ones and by tens.
- **K.CC.2** - Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

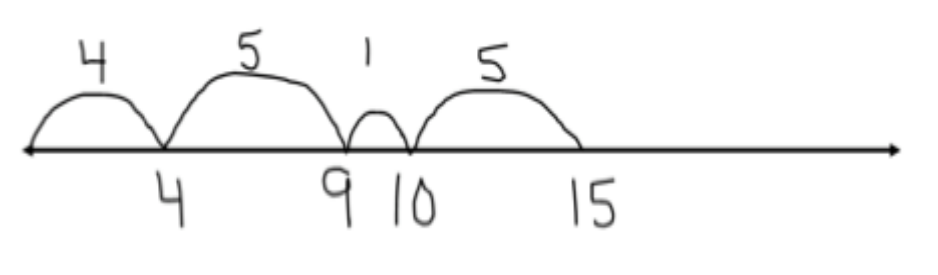
How is the number line helping students make sense of numbers in the following clip?

1st Grade CC Standards

- 1. OA.2 - Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20

- There are cookies on the plate. There are 4 oatmeal raisin cookies, 5 chocolate chip cookies, and 6 gingerbread cookies. How many cookies are there total?

- I used a number line. First I jumped to 4, and then I jumped 5 more. That's 9. Then I jumped 1 more to make 10. Then, I jumped 5 more and got 15. There are 15 cookies on the plate.



2nd Grade CC Standards

Number lines first appear in 2nd grade standards where students are expected to represent numbers as lengths on a number line and to represent sums and differences on a number line.

2.MD.2

- Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

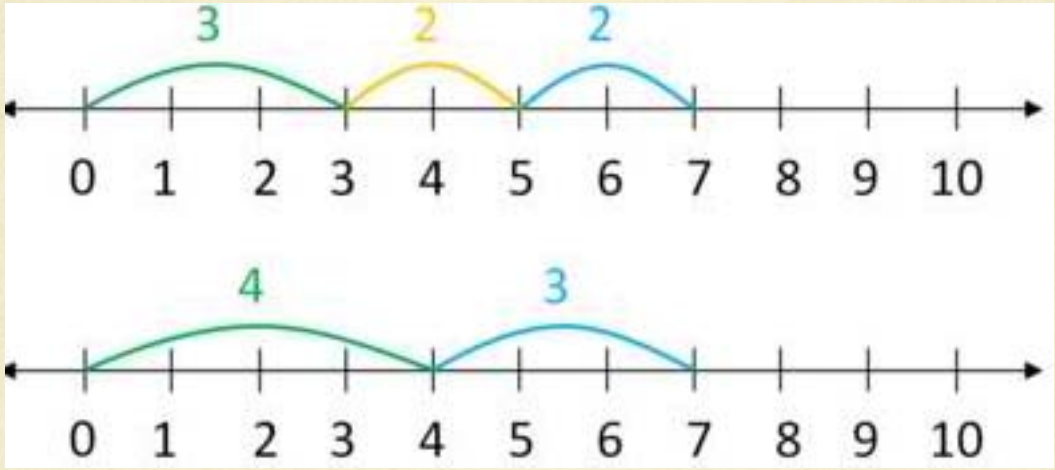
Frog and Toad on a Number Line

- One day, Frog and Toad were sitting together on a lily pad. Some lily pads were in a line across the pond.
- In the morning, Frog hopped three lily pads away. In the afternoon, he hopped two more away. In the evening, he hopped another two more.
- Toad hopped four lily pads away in the morning. He rested in the afternoon and continued three further in the evening. Frog said,
- *Toad, we ended up at the same place!*
  - Show each of their journeys on a number line, starting at 0
  - Use different colors for the morning, afternoon, and evening hops. Write a number sentence that reflects that they ended up at the same place.

Illustrative Mathematics

Solution: Sample solution

- Frog's journey is shown on the top number line and Toad's journey is shown on the bottom number line:

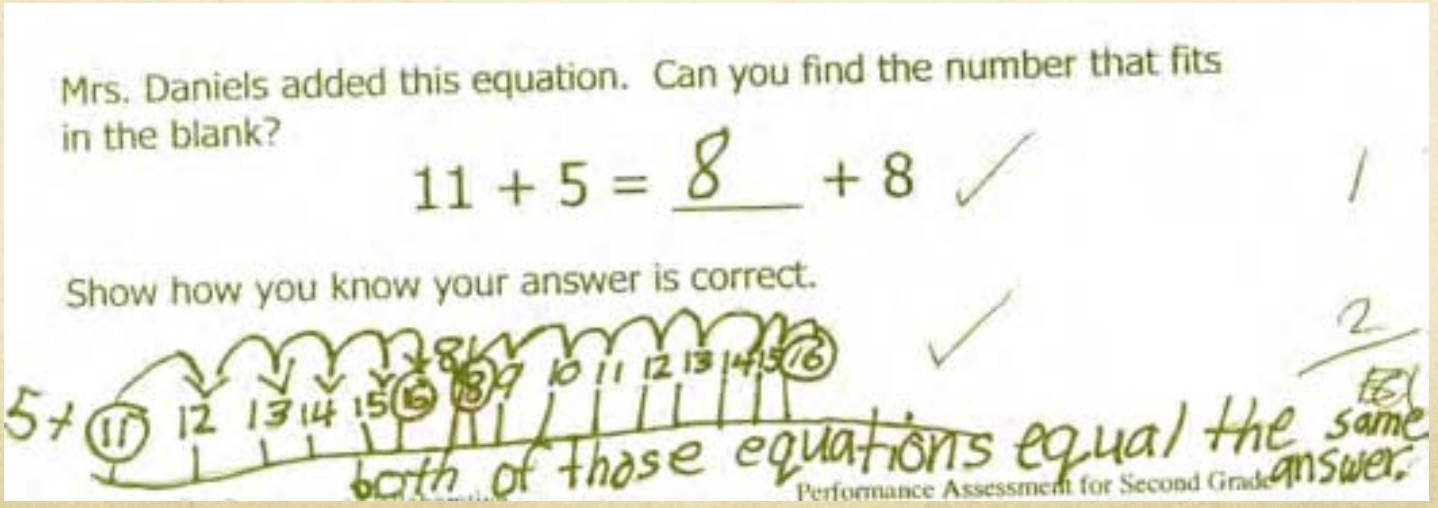


- The number sentence 3 + 2 + 2 = 4 + 3 shows that Frog and Toad end up at the same place.

2nd Grade Example

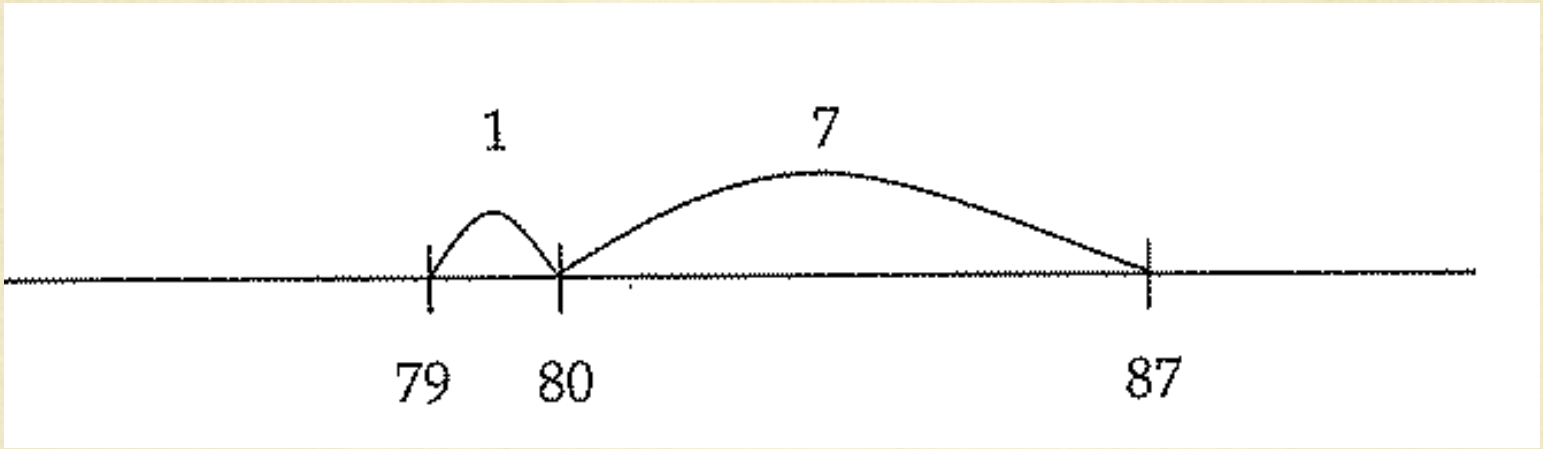
Operations and Algebraic Thinking

- Add and subtract within 20.
- 2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.



Composing and Decomposing Numbers

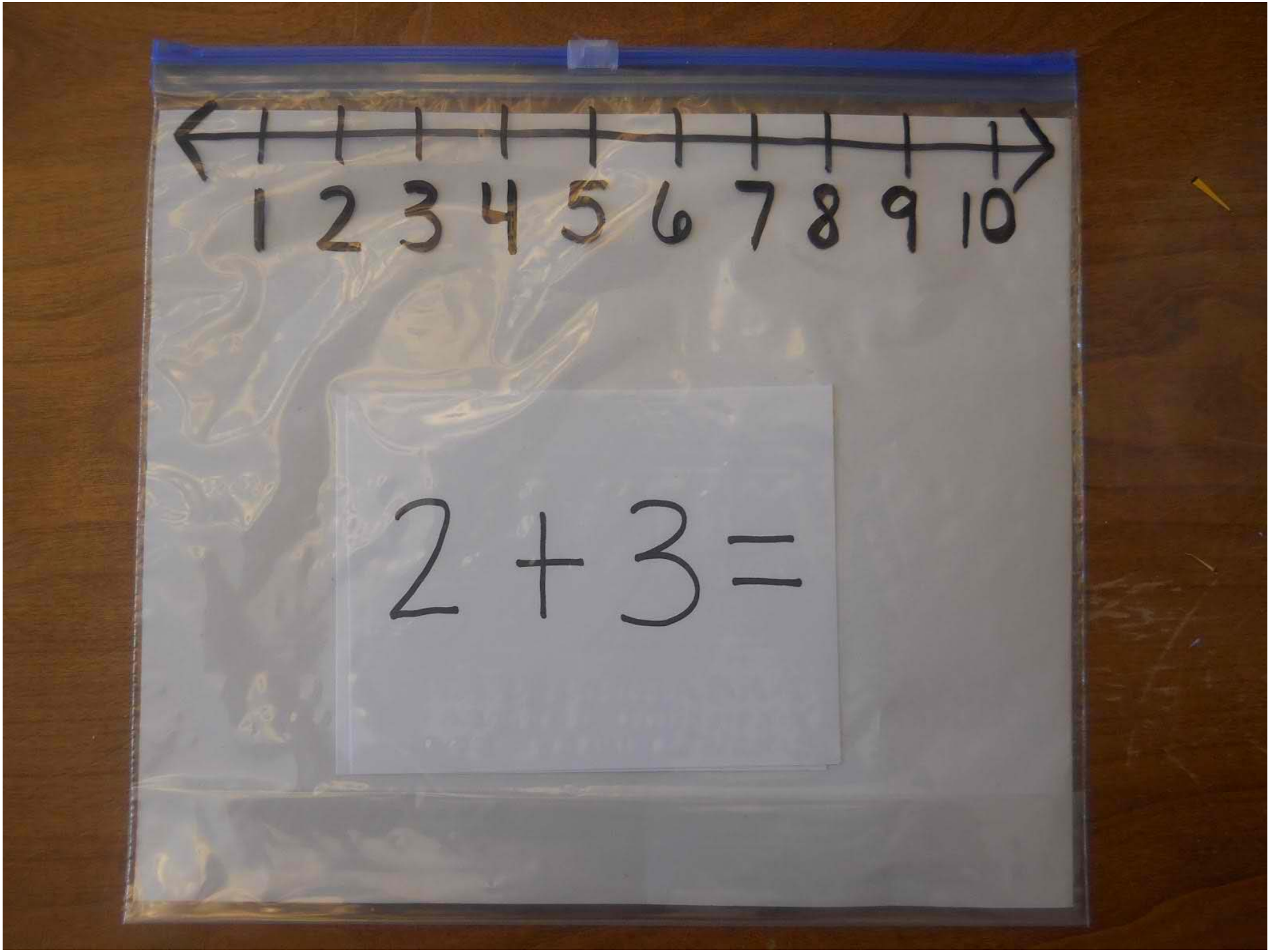
$$79 + 8 = 79 + 1 + 7$$



Additional Activities

- Count a group of objects in the classroom (rulers, windows, shoes, chairs, glue, pencils, noses).
  - Draw a picture of the things counted and clip the picture to the number.
  - Fill a bag with the same number of linking cubes and attach the bag to the number line.
  - Color graph paper to show the number in tens, ones and attach to the number line.
- Arrange number tiles to make a portion of the number line, one student rearranges the number tiles and the other child puts them back in order.
- Place index cards with a variety of numbers under seats for students to add to number line.


Counting
Shapes
1, 2, 3
Illustrated by
Counting
Shapes
1, 2, 3
Illustrated by
Counting
Shapes
1, 2, 3
Illustrated by
Counting
Shapes
1, 2, 3
Illustrated by prathaM
1 2 3 4 5 6 7 8 9 10


1 2 3 4 5 6 7 8 9 10
2 + 3 =


-3
1
-1
-3

With ALL Students

ELL

- Before working with the number line, review key words such as “forward,” “between, “and “greater.”

- Encourage students to write the words in their math journals and have them add illustrations or diagrams to help them remember the meaning of each word.

Reteach/Extra Support

- Gather a small group of students and pre-teach the concept before introducing it with the whole class. Use fewer numbers on the number line or focus on lower numbers.

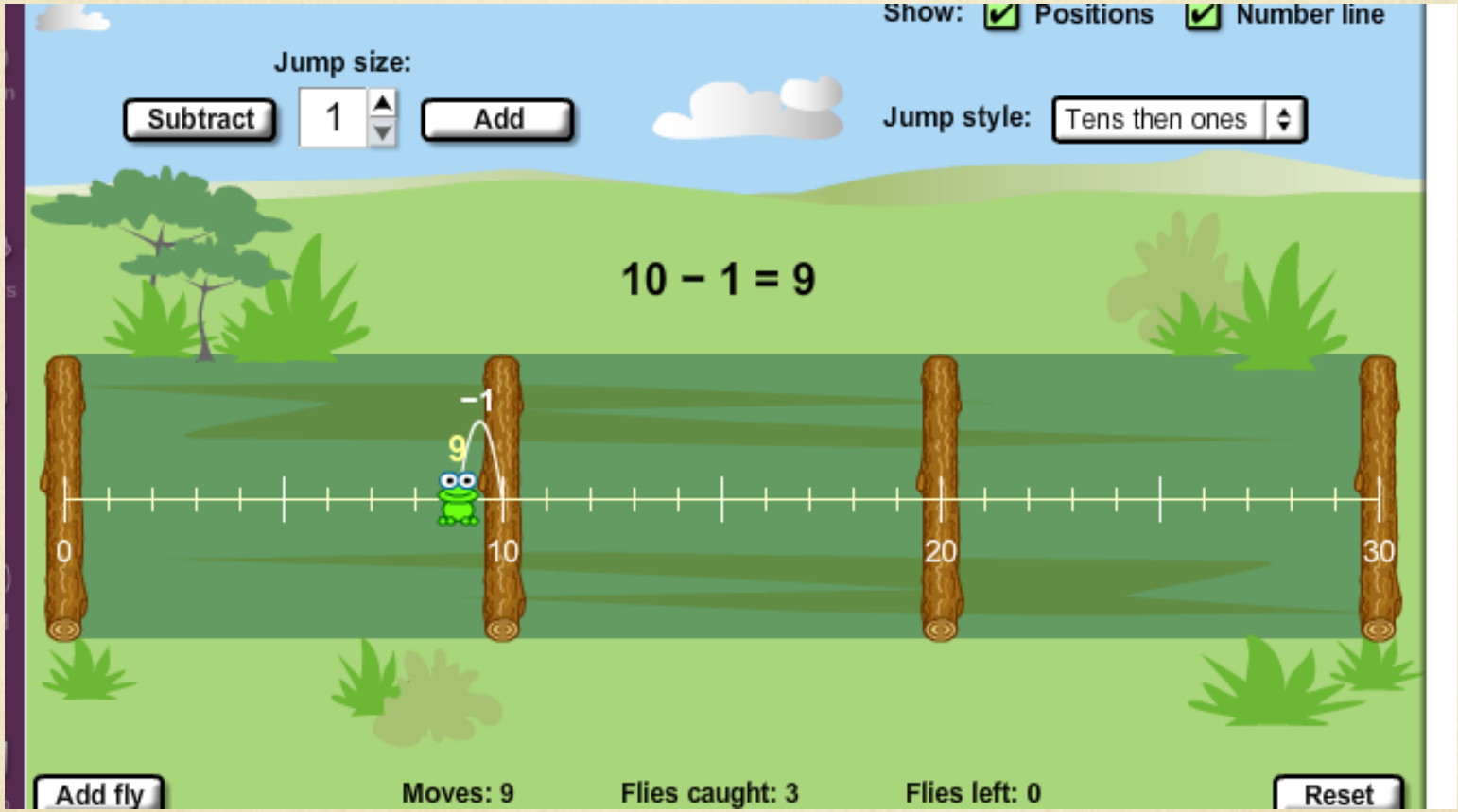
- Invite students to work with partners. Have one child read the activity cards while the other child moves the frog. Then, have the partners switch roles.

With ALL Students

Challenge

- Encourage students to draw their own number lines, extending the numbers above 20.
- Prompt them to write addition, subtraction, sequencing, and comparison problems on index cards. Ask students to exchange problems with other students and use their number lines to solve them.
- Write story problems that can be solved using the number line.

Don't Forget Technology

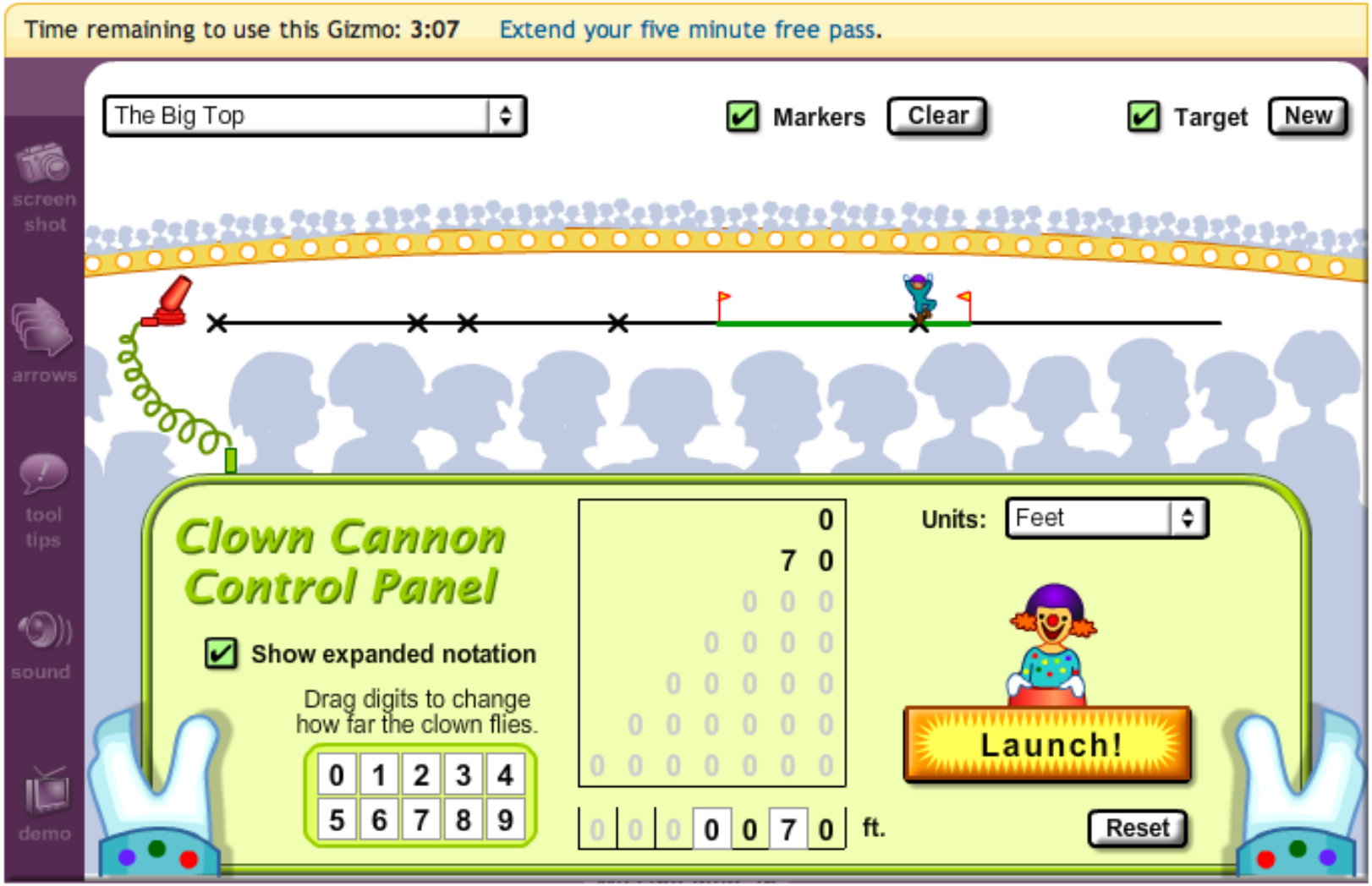


Cannonball Clowns (Number Line Estimation)

Launch clowns from a circus cannon and try to hit the target. Drag digit cards on the control panel to set the launch distance and choose an appropriate unit of distance. After practicing your clown-launching skills on a number line, move on to the Big Top, Football Field, School Buses, the Golden Gate Bridge, and more!

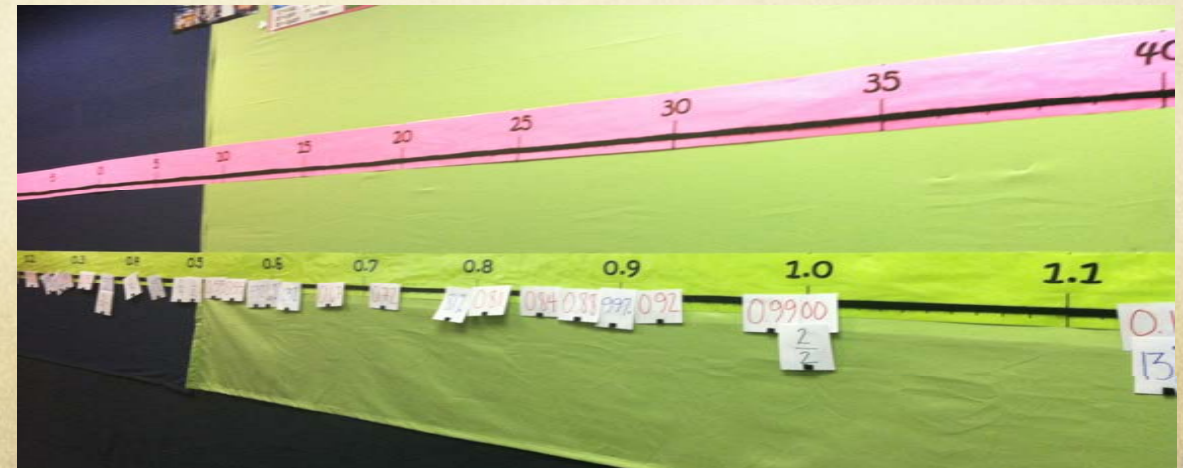
Lesson Info | Gizmo

Lesson Materials | Standard Gizmo Features



Next Steps

- Name three things you learned today that you can implement in your classroom?
- How has your understanding of the importance of the number line changed?
- How will you increase the use of the number line to assist with the transition to the Common Core goal of developing conceptual understanding?



Resources

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Resources

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Resources

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